126 GeV Higgs boson and universality relations in the $SO(5) \times U(1)$ gauge-Higgs unification

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The Higgs boson mass $m_H = 126\,\mathrm{GeV}$ in the $SO(5) \times U(1)$ gauge-Higgs unification in the Randall-Sundrum space leads to important consequences. An universal relation is found between the Kaluza-Klein (KK) mass scale m_{KK} and the Aharonov-Bohm phase θ_H in the fifth dimension; $m_{KK} \sim 1350\,\mathrm{GeV}/(\sin\theta_H)^{0.787}$. The cubic and quartic self-couplings of the Higgs boson become smaller than those in the SM, having universal dependence on θ_H. The decay rates $H \to \gamma\gamma, gg$ are evaluated by summing contributions from KK towers. Corrections coming from KK excited states turn out very small. With $\theta_H = 0.1 \sim 0.35$, the mass of the first KK Z is predicted to be $2.5 \sim 6\,\mathrm{TeV}$.

I. INTRODUCTION

The discovery of a Higgs-like boson with $m_H = 126\,\mathrm{GeV}$ at LHC may give a hint for extra dimensions. We show [1] that the observed Higgs boson mass in the gauge-Higgs unification scenario leads to universal relations among the AB phase θ_H, the KK mass m_{KK}, the Higgs self couplings, and the KK Z boson mass $m_{Z^{(1)}}$, independent of the details of the model.

The gauge-Higgs unification scenario is predictive. As a result of the Hosotani mechanism [2–6] the Higgs boson mass emerges at the quantum level without being afflicted with divergence. The Higgs couplings to the KK towers of quarks and W/Z bosons have a distinctive feature that their signs alternate in the KK level, significant departure from other extra dimensional models such as UED models. As a consequence contributions of KK modes to the decay rate $\Gamma(H \to \gamma\gamma)$ turn out very small. Surprisingly the gauge-Higgs unification gives nearly the same phenomenology at low energies as the standard model (SM).

The gauge-Higgs unification can be confirmed by finding the KK Z boson in the range $2.5 \sim 6\,\mathrm{TeV}$ and by determining the Higgs self couplings and Yukawa couplings at LHC and ILC.

II. $SO(5) \times U(1)$ GAUGE-HIGGS UNIFICATION IN RS

The model is given by $SO(5) \times U(1)$ gauge theory in the Randall-Sundrum (RS) warped space

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \tag{1}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$, $\sigma(y) = \sigma(y+2L) = \sigma(-y)$, and $\sigma(y) = k|y|$ for $|y| \le L$. The RS space is viewed as bulk AdS space $(0 < y < L)$ with AdS curvature $-6k^2$ sandwiched by the Planck brane at $y = 0$ and the TeV brane at $y = L$. The $SO(5) \times U(1)$ model was proposed by Agashe et al [7, 8]. It has been elaborated in refs. [9, 10], and a concrete realistic model has been formulated in ref. [1]. The schematic view of the gauge-Higgs unification is given below.

$$\boxed{\text{5D } A_M} \left\{ \begin{array}{l} \boxed{\text{four-dim. components } A_\mu} \;\in\; \boxed{\text{4D gauge fields } \gamma, W, Z} \\[2em] \boxed{\text{extra-dim. component } A_y} \;\in\; \boxed{\text{4D Higgs field } H} \end{array} \right.$$

$$\sim \boxed{\text{AB phase } \theta_H} \text{ in extra dim.}$$

Hosotani mechanism $\Downarrow$

<u>Dynamical EW symmetry breaking</u>

The 5D Lagrangian density consists of

$$\mathcal{L} = \mathcal{L}_{\rm bulk}^{\rm gauge}(A,B) + \mathcal{L}_{\rm bulk}^{\rm fermion}(\Psi_a, \Psi_F, A, B) + \mathcal{L}_{\rm brane}^{\rm fermion}(\hat{\chi}_\alpha, A, B) + \mathcal{L}_{\rm brane}^{\rm scalar}(\hat{\Phi}, A, B) + \mathcal{L}_{\rm brane}^{\rm int}(\Psi_a, \hat{\chi}_\alpha, \hat{\Phi}) ~. \tag{2}$$

$SO(5)$ and $U(1)_X$ gauge fields are denoted by A_M and B_M, respectively. The two associated gauge coupling constants are g_A and g_B. Two quark multiplets and two lepton multiplets Ψ_a are introduced in the vector representation of $SO(5)$ in each generation, whereas n_F extra fermion multiplets Ψ_F are introduced in the spinor representation. These bulk fields obey the orbifold boundary conditions at $y_0 = 0$ and $y_1 = L$ given by

$$\begin{aligned}
&\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, y_j + y) P_j^{-1}, \\
&\begin{pmatrix} B_\mu \\ B_y \end{pmatrix}(x, y_j - y) = \begin{pmatrix} B_\mu \\ -B_y \end{pmatrix}(x, y_j + y) ~, \\
&\Psi_a(x, y_j - y) = P_j \Gamma^5 \Psi_a(x, y_j + y) ~, \quad \Psi_F(x, y_j - y) = (-1)^j P_j^{\rm sp} \Gamma^5 \Psi_F(x, y_j + y) ~, \\
&P_j = {\rm diag}\,(-1,-1,-1,-1,1) ~, \quad P_j^{\rm sp} = {\rm diag}\,(1,1,-1,-1) ~.
\end{aligned} \tag{3}$$

The orbifold boundary conditions break $SO(5) \times U(1)_X$ to $SO(4) \times U(1)_X \simeq SU(2)_L \times SU(2)_R \times U(1)_X$.

The brane interactions are invariant under $SO(4) \times U(1)_X$. The brane scalar $\hat{\Phi}$ is in the $(\mathbf{1}, \mathbf{2})_{-1/2}$ representation of $[SU(2)_L, SU(2)_R]_{U(1)_X}$. It spontaneously breaks $SU(2)_R \times U(1)_X$ to $U(1)_Y$ by non-vanishing $\langle \hat{\Phi} \rangle$ whose magnitude is supposed to be much larger than the KK scale $m_{\rm KK}$. At this stage the residual gauge symmetry is $SU(2)_L \times U(1)_Y$. Brane fermions $\hat{\chi}_\alpha$ are introduced in the $(\mathbf{2}, \mathbf{1})$ representation. The quark-lepton vector multiplets Ψ_a are decomposed into $(\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$. The $(\mathbf{2}, \mathbf{2})$ part of Ψ_a, $\hat{\chi}_\alpha$ in $(\mathbf{2}, \mathbf{1})$ and $\hat{\Phi}$ in $(\mathbf{1}, \mathbf{2})$ form $SO(4) \times U(1)_X$ invariant brane interactions. With $\langle \hat{\Phi} \rangle \neq 0$ they yield mass terms. The resultant spectrum of massless fermions is the same as in the SM. All exotic fermions become heavy, acquiring masses of $O(m_{\rm KK})$. Further with brane fermions all anomalies associated with gauge fields of $SO(4) \times U(1)_X$ are cancelled.[10]

With the orbifold boundary conditions (3) there appear four zero modes of A_y in the components $(A_y)_{a5} = -(A_y)_{5a}$ $(a = 1, \cdots, 4)$. They form an $SO(4)$ vector, or an $SU(2)_L$ doublet, corresponding to the Higgs doublet in the SM. The AB phase is defined with these zero modes by

$$e^{i\Theta_H/2} \sim P \exp \left\{ i g_A \int_0^L dy\, A_y \right\} ~. \tag{4}$$

At the tree level the value of the AB phase Θ_H is not determined, as it gives vanishing field strengths. At the quantum level its effective potential $V_{\rm eff}$ becomes non-trivial. The value of Θ_H is determined by the location of the minimum of $V_{\rm eff}$. This is the Hosotani mechanism and induces dynamical gauge symmetry breaking. It leads to gauge-Higgs unification, resolving the gauge-hierarchy problem.[6] Without loss of generality one can assume that $(A_y)_{45}$ component develops a non-vanishing expectation value. Let us denote the corresponding component of Θ_H by θ_H. If θ_H takes a non-vanishing value, the electroweak symmetry breaking takes place.

III. $V_{\rm eff}(\theta_H)$ AND m_H

Given the matter content one can evaluate $V_{\rm eff}(\theta_H)$ at the one loop level unambiguously. The θ_H dependent part of $V_{\rm eff}(\theta_H)$ is finite, being free from divergence. $V_{\rm eff}(\theta_H)$ depends on several parameters of the theory; $V_{\rm eff} = V_{\rm eff}(\theta_H; \xi, c_t, c_F, n_F, k, z_L)$ where ξ is the gauge parameter in the generalized R_ξ gauge, c_t and c_F are the bulk mass parameters of the top and extra fermion multiplets, n_F is the number of the extra fermion multiplets, and k, z_L are parameters specifying the RS metric (1). Given these parameters, $V_{\rm eff}$ is fixed, and the location of the global minimum of $V_{\rm eff}(\theta_H)$, $\theta_H^{\rm min}$, is determined.

With $\theta_H^{\rm min}$ determined, m_Z, g_w, $\sin^2\theta_W$ are determined from g_A, g_B, k, z_L and $\theta_H^{\rm min}$. The top mass m_t is determined from $c_t, k, z_L, \theta_H^{\rm min}$, whereas the Higgs boson mass m_H is given by

$$m_H^2 = \frac{1}{f_H^2} \frac{d^2 V_{\rm eff}}{d\theta_H^2}\bigg|_{\rm min}, \qquad f_H = \frac{2}{g_w} \sqrt{\frac{k}{L(z_L^2 - 1)}} ~. \tag{5}$$

Let us take $\xi = 1$. Then the theory has seven parameters $\{g_A, g_B, k, z_L, c_t, c_F, n_F\}$. Adjusting theses parameters, we reproduce the values of five observed quantities $\{m_Z, g_w, \sin^2\theta_W, m_t, m_H\}$. This leaves two parameters, say z_L and n_F, free. Put differently, the value of $\theta_H^{\rm min}$ is determined as a function of z_L and n_F; $\theta_H^{\rm min} = \theta_H(z_L, n_F)$. We comment that contributions from other light quark/lepton multiplets to $V_{\rm eff}$ are negligible.

$V_{\rm eff}(\theta_H)$ in the absence of the extra fermions ($n_F = 0$) was evaluated in refs. [9, 11]. It was found there that the global minima naturally appear at $\theta_H = \pm\frac{1}{2}\pi$ at which the Higgs boson becomes absolutely stable. It is due to the emergence of the H parity invariance.[11, 12] In particular the Higgs trilinear couplings to W, Z, quarks and leptons are all proportional to $\cos\theta_H$ and vanish at $\theta_H = \pm\frac{1}{2}\pi$.[13–18]

This, however, conflicts with the observation of an unstable Higgs boson at LHC. To have an unstable Higgs boson the H parity invariance must be broken, which is most easily achieved by introducing extra fermion multiplets Ψ_F in the spinor representation of $SO(5)$ in the bulk.[1]

Let us take $n_F = 3, z_L = e^{kL} = 10^7$ as an example. $\{g_w, \sin^2\theta_W\}$ are related to $\{g_A, g_B\}$ by

$$g_w = \frac{g_A}{\sqrt{L}} \quad , \quad \tan\theta_W = \frac{g_B}{\sqrt{g_A^2 + g_B^2}} \quad , \tag{6}$$

where $z_L = e^{kL}$. The observed values of $\{m_Z, g_w, \sin^2\theta_W, m_t, m_H\}$ are reproduced with $k = 1.26 \times 10^{10}\,$GeV, $c_t = 0.330$, $c_F = 0.353$ for which the minima of $V_{\rm eff}$ are found at $\theta_H = \pm 0.258$. The KK mass scale is $m_{\rm KK} = \pi k z_L^{-1} = 3.95\,$TeV. $V_{\rm eff}(\theta_H)$ is depicted in Fig. 1 with red curves. For comparison $V_{\rm eff}$ in the case of $n_F = 0$ is also plotted with a blue curve. When $n_F = 0$ and $z_L = 10^7$, the minima are located at $\theta_H = \pm\frac{1}{2}\pi$. The observed values of $\{m_Z, g_w, \sin^2\theta_W, m_t\}$ are reproduced with $k = 3.16 \times 10^9\,$GeV and $c_t = 0.345$. In this case the Higgs boson mass determined by (5) becomes $m_H = 87.9\,$GeV, and $m_{\rm KK} = 993\,$GeV. One can see how the position of the minima is shifted from $\theta_H = \pm\frac{1}{2}\pi$ to $\theta_H = \pm 0.082\pi = \pm 0.258$ by the introduction of the extra fermions.

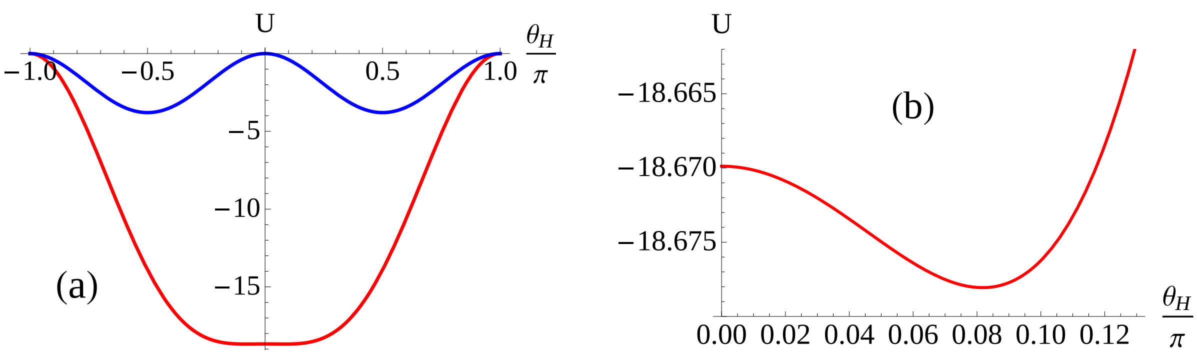

FIG. 1: The effective potential $V_{\rm eff}(\theta_H)$ for $z_L = 10^7$. $U = 16\pi^6 m_{\rm KK}^{-4} V_{\rm eff}$ is plotted. The red curves are for $n_F = 3$ with $m_H = 126\,$GeV. $V_{\rm eff}$ has minima at $\theta_H = \pm 0.258$ and $m_{\rm KK} = 3.95\,$TeV. The blue curve is for $n_F = 0$ in which case $m_H = 87.9\,$GeV and $m_{\rm KK} = 993\,$GeV.

IV. UNIVERSALITY

As explained above, the AB phase $\theta_H (= \theta_H^{\rm min})$ is determined as a function of z_L and n_F; $\theta_H(z_L, n_F)$. The KK mass scale $m_{\rm KK} = \pi k z_L^{-1}$ is also determined as a function of z_L and n_F; $m_{\rm KK}(z_L, n_F)$. The relation between them is plotted for $n_F = 1, 3, 9$ in the top figure in Fig. 2. One sees that all points fall on one universal curve to good accuracy, independent of n_F.

Similarly one can evaluate the cubic (λ_3) and quartic (λ_4) self-couplings of the Higgs boson H by expanding $V_{\rm eff}[\theta_H + (H/f_H)]$ around the minimum in a power series in H. They are depicted in the bottom figure in Fig. 2. Although the shape of $V_{\rm eff}(\theta_H)$ heavily depends on n_F, the relations $\lambda_3(\theta_H)$ and $\lambda_4(\theta_H)$ turn out universal, independent of n_F.

It is rather surprising that there hold universal relations among θ_H, $m_{\rm KK}$, λ_3 and λ_4. Once θ_H is determined from one source of observation, then many other physical quantities are fixed and predicted. The gauge-Higgs unification gives many definitive predictions to be tested by experiments. We tabulate values of various quantities determined from $m_H = 126\,$GeV with given z_L for $n_F = 3$ in Table I. The relation between θ_H and

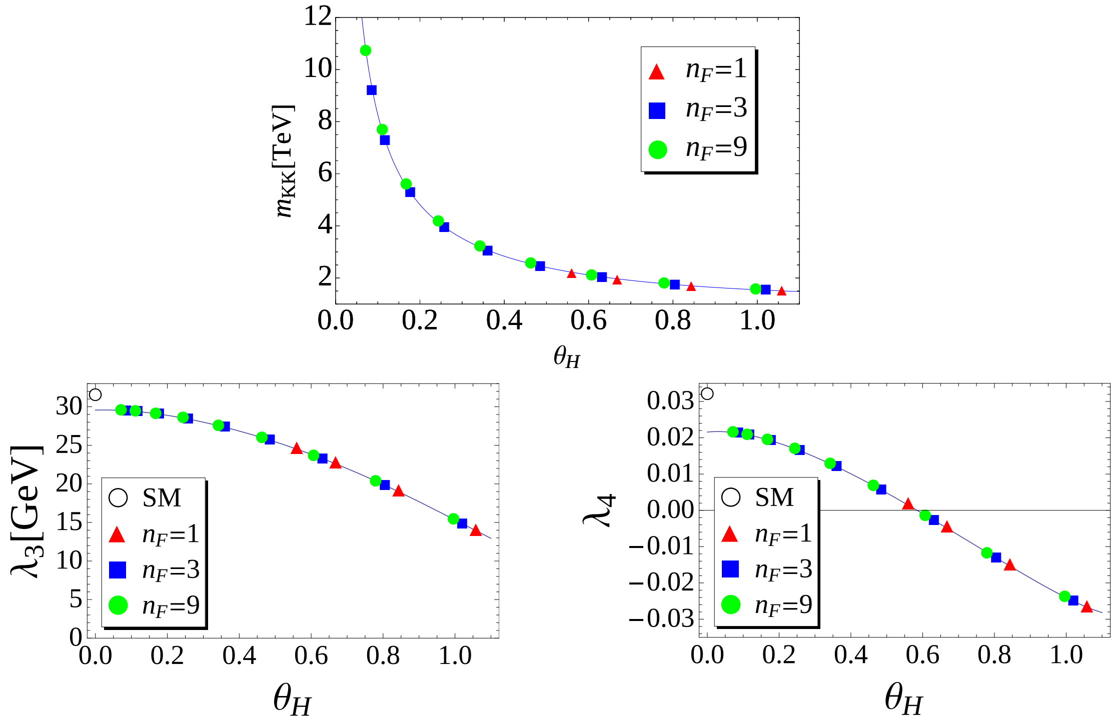

FIG. 2: Universality relations. [Top] KK scale $m_{\rm KK}(\theta_H)$. [Bottom left] Higgs cubic self-coupling $\lambda_3(\theta_H)$. [Bottom right] Higgs quartic self-coupling $\lambda_4(\theta_H)$. The universality, independent of n_F, is seen in all relations.

$m_{\rm KK}$ is well summarized with

$$m_{\rm KK} \sim \frac{1350\,{\rm GeV}}{(\sin\theta_H)^{0.787}} \ . \tag{7}$$

TABLE I: Values of the various quantities with given z_L for $n_F = 3$. $m_{Z^{(1)}}$ and $m_{F^{(1)}}$ are masses of the first KK Z boson and the lowest mode of the extra fermion multiplets. Relations among θ_H, $m_{\rm KK}$ and $m_{Z^{(1)}}$ are universal, independent of n_F.

z_L	θ_H	$m_{\rm KK}$	$m_{Z^{(1)}}$	$m_{F^{(1)}}$
10^8	0.360	3.05 TeV	2.41 TeV	0.668 TeV
10^7	0.258	3.95	3.15	0.993
10^6	0.177	5.30	4.25	1.54
10^5	0.117	7.29	5.91	2.53

V. $H \to \gamma\gamma, gg$

In the gauge-Higgs unification all of the 3-point couplings of W, Z, quarks and leptons to the Higgs boson H at the tree level are suppressed by a common factor $\cos\theta_H$ compared with those in the SM.[13–18] The decay of the Higgs boson to two photons goes through loop diagrams in which W boson, quarks, leptons, extra fermions and their KK excited states run.

The decay rate $\Gamma[H \to \gamma\gamma]$ is given by

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2 g_w^2}{1024\pi^3} \frac{m_H^3}{m_W^2} \left|\mathcal{F}_{\rm total}\right|^2 ,$$

$$
\begin{aligned}
&\mathcal{F}_{\rm total} = \mathcal{F}_W + \frac{4}{3}\mathcal{F}_{\rm top} + \Big(2(Q_X^{(F)})^2 + \tfrac{1}{2}\Big)\, n_F \mathcal{F}_F ~, \\
&\mathcal{F}_W = \cos\theta_H \sum_{n=0}^\infty I_{W^{(n)}} \frac{m_W}{m_{W^{(n)}}}\, F_1(\tau_{W^{(n)}}) ~,~~ I_{W^{(n)}} = \frac{g_{HW^{(n)}W^{(n)}}}{g_w m_{W^{(n)}} \cos\theta_H} ~, \\
&\mathcal{F}_{\rm top} = \cos\theta_H \sum_{n=0}^\infty I_{t^{(n)}} \frac{m_t}{m_{t^{(n)}}}\, F_{1/2}(\tau_{t^{(n)}}) ~,~~ I_{t^{(n)}} = \frac{y_{t^{(n)}}}{y_t^{\rm SM} \cos\theta_H} ~, \\
&\mathcal{F}_F = \sin\tfrac{1}{2}\theta_H \sum_{n=1}^\infty I_{F^{(n)}} \frac{m_t}{m_{F^{(n)}}}\, F_{1/2}(\tau_{F^{(n)}}) ~,~~ I_{F^{(n)}} = \frac{y_{F^{(n)}}}{y_t^{\rm SM} \sin\frac{1}{2}\theta_H} ~,
\end{aligned}
\tag{8}
$$

where $W^{(0)} = W$, $t^{(0)} = t$, $\tau_a = 4m_a^2/m_H^2$. The functions $F_1(\tau)$ and $F_{1/2}(\tau)$ are defined in Ref. [19], and $F_1(\tau) \sim 7$ and $F_{1/2}(\tau) \sim -\frac{4}{3}$ for $\tau \gg 1$. $Q_X^{(F)}$ is the $U(1)_X$ charge of the extra fermions. $I_{W^{(0)}}$ and $I_{t^{(0)}}$ are ~ 1.

In Fig. 3, $I_{W^{(n)}}$, $I_{t^{(n)}}$, and $I_{F^{(n)}}$ are plotted. One sees that the values of these I's alternate in sign as n increases, which gives sharp contrast to the UED models.

$$
I_{W^{(n)}} \sim (-1)^n I_W^\infty ~~,~~ I_{t^{(n)}} \sim (-1)^n I_t^\infty ~~,~~ I_{F^{(n)}} \sim (-1)^n I_F^\infty \qquad \text{for } n \gg 1 \tag{9}
$$

up to $(\ln n)^p$ corrections. This is special to the gauge-Higgs unification models. It has been known in the models in flat space as well.[20, 21] As a consequence of the destructive interference due to the alternating sign, the infinite sums in the rate (8) converges rapidly. There appears no divergence.

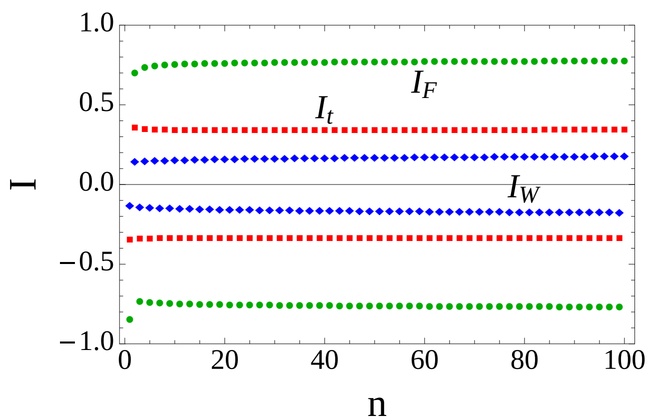

FIG. 3: $I_{W^{(n)}}$, $I_{t^{(n)}}$, and $I_{F^{(n)}}$ for $n_F = 3$, $Q_X^{(F)} = 0$ and $\theta_H = 0.360$ ($z_L = 10^8$) in the range $1 \le n \le 100$. $I_{W^{(0)}} = 1.004$ and $I_{t^{(0)}} = 1.012$.

Let $\mathcal{F}_{W\,\rm only}$ and $\mathcal{F}_{t\,\rm only}$ be the contributions of $W = W^{(0)}$ and $t = t^{(0)}$ to $\mathcal{F}_{\rm total}$. The numerical values of the amplitudes $\mathcal{F}$'s are tabulated in Table II for $n_F = 3$. It is seen that contributions of KK states to the amplitude are small. The dominant effect for the decay amplitude is the suppression factor $\cos\theta_H$.

All Higgs couplings $HWW, HZZ, Hc\bar{c}, Hb\bar{b}, H\tau\bar{\tau}$ are suppressed by a factor $\cos\theta_H$ at the tree level. The corrections to $\Gamma[H \to \gamma\gamma]$ and $\Gamma[H \to gg]$ due to KK states amount only to 0.2% (2%) for $\theta_H = 0.117(0.360)$. Hence we conclude

$$
\begin{aligned}
&\text{branching fraction:} \quad B(H \to j) \sim B^{\rm SM}(H \to j) \\
&\qquad\qquad\qquad\qquad\qquad j = WW, ZZ, \gamma\gamma, gg, b\bar{b}, c\bar{c}, \tau\bar{\tau}, \cdots \\
&\gamma\gamma \text{ production rate:} \quad \sigma^{\rm prod}(H) \cdot B(H \to \gamma\gamma) \sim ({\rm SM}) \times \cos^2\theta_H ~.
\end{aligned}
\tag{10}
$$

The signal strength in the $\gamma\gamma$ production relative to the SM is about $\cos^2\theta_H$. It is about 0.99 (0.91) for $\theta_H = 0.1$ (0.3). This contrasts to the prediction in the UED models in which the contributions of KK states can add up in the same sign to sizable amount.[22]

TABLE II: Values of the amplitudes $\mathcal{F}$'s in (8) for $n_F = 3$ and $Q_X^{(F)} = 0$.

θ_H	0.117	0.360
z_L	10^5	10^8
$\mathcal{F}_{W\,\mathrm{only}}$	8.330	7.873
$\mathcal{F}_W/\mathcal{F}_{W\,\mathrm{only}}$	0.9996	0.998
$\mathcal{F}_{t\,\mathrm{only}}$	-1.372	-1.305
$\mathcal{F}_t/\mathcal{F}_{t\,\mathrm{only}}$	0.998	0.990
$\mathcal{F}_F/\mathcal{F}_{t\,\mathrm{only}}$	-0.0034	-0.033
$\mathcal{F}_{\mathrm{total}}$	6.508	6.199
$\mathcal{F}_{\mathrm{total}}/(\mathcal{F}_{W\,\mathrm{only}} + \frac{4}{3}\mathcal{F}_{t\,\mathrm{only}})$	1.001	1.011

VI. SIGNALS OF GAUGE-HIGGS UNIFICATION

There are several constraints to be imposed on the gauge-Higgs unification.

(i) For the consistency with the S parameter, we need $\sin\theta_H < 0.3$.[7]

(ii) The tree-level unitarity requires $\theta_H < 0.5$.[23]

(iii) Z' search at Tevatron and LHC. The first KK Z corresponds to Z'. No signal has been found so far, which implies that $m_{Z^{(1)}} > 2\,\mathrm{TeV}$. With the universality relations in Sec. IV it requires $\theta_H < 0.4$.

(iv) In ref. [24] the consistency with other precision measurements such as the Z boson decay and the forward-backward asymmetry on the Z resonance has been investigated when $n_F = 0$. Reasonable agreement was found for $m_{\mathrm{KK}} > 1.5\,\mathrm{TeV}$. We need to reanalyze in the case $n_F \geq 1$.

All of those constraints above point $\theta_H < 0.4$. When θ_H is very small, the KK mass scale m_{KK} becomes very large and it becomes very difficult to distinguish the gauge-Higgs unification from the SM. The range of interest is $0.1 < \theta_H < 0.35$, which can be explored at LHC with an increased energy 13 or 14 TeV. The gauge-Higgs unification predicts the following signals.

(1) The first KK Z should be found at $m_{\mathrm{KK}} = 2.5 \sim 6\,\mathrm{TeV}$ for $\theta_H = 0.35 \sim 0.1$.

(2) The Higgs self-couplings should be smaller than those in the SM. λ_3 (λ_4) should be $10 \sim 20\%$ ($30 \sim 60\%$) smaller for $\theta_H = 0.1 \sim 0.35$, according to the universality relations. This should be explored at ILC.

(3) The lowest mode ($F^{(1)}$) of the KK tower of the extra fermion Ψ_F should be discovered at LHC. Its mass depends on both θ_H and n_F. For $n_F = 3$, the mass is predicted to be $m_{F^{(1)}} = 0.7 \sim 2.5\,\mathrm{TeV}$ for $\theta_H = 0.35 \sim 0.1$.

VII. FOR THE FUTURE

The $SO(5) \times U(1)$ gauge-Higgs unification model of ref. [1] has been successful so far. Yet further elaboration may be necessary.

(1) Flavor mixing has to be incorporated to explore flavor physics.[25]

(2) It is curious to generalize the model to incorporate SUSY. The Higgs boson mass becomes smaller than in non-SUSY model. $m_H = 126\,\mathrm{GeV}$ should give information about SUSY breaking scales.[26]

(3) The orbifold boundary conditions (P_0, P_1) in (3) have been given by hand so far. It is desirable to have dynamics which determine the boundary conditions.[27, 28]

(4) Not only electroweak interactions but also strong interactions should be integrated in the form of grand gauge-Higgs unification.[29]

Acknowledgments

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